LINEAR TECHNIQUES APPLIED TO SMALL-SIGNAL ELECTROMECHANICAL STABILITY, MODEL ORDER REDUCTION AND HARMONIC STUDIES

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ABSTRACT—This paper describes recent work on small-signal stability, model order reduction and power system harmonic analysis carried out in CEPEL, the Brazilian Electrical Energy Research Center. CEPEL has developed, along the last three decades, a suite of power system analysis tools that are in current use by most of the Brazilian electrical utilities.

Key Words: Small-Signal Stability, Modal Analysis, Model Reduction, Power System Harmonics.

1. INTRODUCTION
This paper describes linear techniques developed in CEPEL for the analysis of power system dynamic models [1]. The described developments are:
1. Fast computation and effective use of linear time responses and modal time responses of large power system models;
2. Computation of reduced models of high-order multivariable transfer functions;
3. Application of transfer function zeros to better understand and control the adverse transients on generator terminal voltage and reactive power induced by the presence of Power System Stabilizers (PSS);
4. Modal analysis in power system harmonic studies.

Other power system dynamics and control developments, carried out by the authors, are listed in the last section of this paper together with the available references.

2. LINEAR TIME RESPONSES AND MODAL TIME RESPONSES OF LARGE POWER SYSTEMS
The large system model utilized to produce the results of this section was the North-South Brazilian interconnection. The North-Northeast and South-Southeast subsystems were interconnected in 1999 through a 1,000 km long, series-compensated 500 kV transmission line. Thyristor Controlled Series Compensators (TCSCs) were placed at the two ends of this line, in order to damp the North-South mode, a low-frequency, poorly damped interarea oscillation mode associated with this interconnection.

The Brazilian system model for the year 1999 has 2,370 buses, 3,401 lines/transformers, 2,519 voltage dependent loads, 123 synchronous machines, 122 excitation systems, 46 power system stabilizers, 99 speed-governors, 4 static Var compensators, 2 TCSCs equipped with Power Oscillation Damping (POD) controllers, 1 HVDC link with two bipoles. Each synchronous machine and associated controls is the aggregate model of a whole power plant. All system equipment relevant to the study were modeled in detail, and the system Jacobian matrix has 13,062 rows and 48,521 nonzero elements [2]. The system has 1,676 state variables. The North-South mode in the presence of the two TCSCs, is associated with the complex pair of eigenvalues $s = -0.31793 \pm j 1.04375$, having a damping ratio of 29% and frequency of 0.17 Hz, for the operating condition considered. The POD controllers of the two TCSCs are the major source of damping for the North-South mode.
The use of linear time response simulation has proved valuable to the oscillation damping analysis and control of large systems. Obtaining the time response of the linear model by the sparsity-preserving, implicit trapezoidal rule algorithm takes 50 times less CPU time than that needed by current transient stability programs [3].

QR routines from standard mathematical libraries [4, 5], have performed quite reliably for power system state matrices of about 2,000 states, considering practical controller parameters and operating conditions. The CPU time for the QR eigensolution of the 1,676 state matrix is about 3 minutes on a Pentium personal computer.

Figure 1 and Figure 3 show the plots of the QR eigensolutions of the power system state matrix for the North-South Brazilian system without PSSs (1,359 states) and with PSSs (1,676 states) respectively. There are 46 major power plants with their PSSs represented, plus the POD controllers at the two TCSCs. Figure 2 and Figure 4 show the associated linear time responses for step disturbances of different polarities, simultaneously applied to the exciter voltage references or to the mechanical power references of several system generators. The variables being plotted are the rotor speed deviations in selected generators. The system is seen to have several unstable modes in the absence of PSS and POD controllers (Figure 1 and Figure 2) and stable in their presence (Figure 3 and Figure 4), showing they are essential to keep the system operating safely at the current levels of power transfer.

The North-South mode (0.17 Hz) is highly observable in Figure 4. It is quite well damped mostly due to the presence of the TCSC POD controllers as may be inferred from the comparison of Figure 4 and Figure 5. Figure 5 was obtained for a slightly different disturbance applied to basically the same system model, the only change being that the two POD controllers were disabled.
The next two figures illustrate the effectiveness of the model-order reduction techniques available, which are based on dominant poles and associated residues of scalar transfer functions of the full-size system.

Figure 6 compares the rotor speed deviations for one of the generators monitored in Figure 5 (generator #5022) obtained by step-by-step numerical integration on the 1,664-state system with that obtained by Inverse Laplace Transform of a step disturbance applied to the 6th order transfer function modal equivalent. The 6th order model was directly obtained by using the Dominant Pole Spectrum Eigensolver algorithm [6]. The step response of the modal equivalent was produced by computing the 6th order approximation to \( y(t) \) every 20 milliseconds:

\[
y(t) \equiv \sum_{i=1}^{6} \frac{R_i}{\lambda_i} (e^{\lambda_i t} - 1), \\
\]

where:

\[
\begin{align*}
\lambda_{1,2} &= -0.034 \pm 1.079j \quad R_{1,2} = 0.015 \angle 1.3^\circ \\
\lambda_{3,4} &= -2.437 \pm 0.054j \quad R_{3,4} = 0.010 \angle 77^\circ \\
\lambda_{5,6} &= -0.521 \pm 2.881j \quad R_{5,6} = 0.001 \angle 111^\circ \\
\end{align*}
\]

The rotor speed deviations for generator #5022, obtained from the 1,676-state system model, are compared in Figure 7 with the response of a 10th-order modal equivalent. The step response of the modal equivalent was produced by computing the 10th order approximation to \( y(t) \) every 20 milliseconds:

\[
y(t) \equiv \sum_{i=1}^{10} \frac{R_i}{\lambda_i} (e^{\lambda_i t} - 1), \\
\]

where:

\[
\begin{align*}
\lambda_{1,2} &= -7.016 \pm 2.918j \quad R_{1,2} = 0.064 \angle 93^\circ \\
\lambda_{3,4} &= -2.996 \pm 9.390j \quad R_{3,4} = 0.030 \angle 44^\circ \\
\lambda_{5,6} &= -0.318 \pm 1.044j \quad R_{5,6} = 0.022 \angle 7^\circ \\
\lambda_{7,8} &= -0.346 \pm 0.580j \quad R_{7,8} = 0.013 \angle 175^\circ \\
\lambda_{9,10} &= -0.116 \pm 0.245j \quad R_{9,10} = 0.002 \angle 148^\circ \\
\end{align*}
\]

The above results are for a scalar, also referred as Single-Input-Single-Output (SISO), transfer function. Modal equivalents for multivariable, also referred as Multi-Input-Multi-Output (MIMO), transfer

Figure 5. Time response for Brazilian system with 46 power system stabilizers but no POD controllers (1,664 states).
functions are necessarily of a higher order, but can also be obtained at reasonable cost, as described in the next section of this paper.

Another important use for the linear time response function is the validation of the small-signal analysis results for large systems. This involves verifying whether the results obtained with a benchmark transient stability program, for a sufficiently small disturbance, match those produced by the small-signal program. Figure 8 and Figure 9 show that, for a small step disturbance simultaneously applied to several generators, the responses from linear and non-linear simulations are in agreement. The monitored variable is the deviations in active power flow in the North-South tie-line.

Figure 8. Comparison of Linear and Non-Linear System Response for a Small Disturbance. System without PSSs in Paulo Afonso IV and Xingó Power Plants and no POD controllers (100 MVA Base).

Figure 9. Comparison of Linear and Non-Linear System Response for a Small Disturbance. System Model with Proposed PSSs in Paulo Afonso IV and Xingó Power Plants (100 MVA Base).

3. REDUCED MODELS OF HIGH-ORDER MULTIVARIABLE TRANSFER FUNCTIONS

Model reduction is an important step to many control systems applications. Highly reduced models for power system transfer functions, either scalar (SISO) or multivariable (MIMO), proved effective for controller design [4, 7, 8, 9, 10]. The model reduction algorithms applied to power systems have been both model-based [6, 9] and measurement-based [7, 8]. An \(m \times m\) transfer function \(G(s)\) of an \(n\)-th order system, having a complete set of eigenvectors, can be expressed as (1).

\[
G(s) = \sum_{i=1}^{n} \frac{R_i}{s - \lambda_i} \tag{1}
\]

The above expression for \(G(s)\), involves the sum of \(m \times m\) residue matrices over a first-order pole [11]. The symbol \(R_i\) denotes the residue matrix for \(G(s)\) at the pole \(\lambda_i\); it may be computed once the eigentriplet \((\lambda_i, x_i, v_i)\) has been obtained [12]. An efficient eigensolution algorithm [13] was recently developed to only find the most dominant poles of high-order multivariable transfer functions. Adequate use of this algorithm allows effective model-based system reduction.

The Sigma plot [13] of the reduced-order model (2) should be evaluated over a sufficiently large number of ‘\(\omega\)’ values within the dynamic range of \(G(s)\). Note that the order of the reduced model is \(p = 2nc + nr\), where \(nc\) is the number of complex-conjugate eigenvalue pairs and \(nr\) the number of real eigenvalues retained in the model.

\[
G(s) \approx \sum_{i=1}^{p} \frac{R_i}{j\omega - \lambda_i} \tag{2}
\]

The North-South Brazilian system, described in the previous section, was again utilized here. An \(8 \times 8\) transfer function matrix \(G(s)\), relating variables from 8 power plants located in the Northeastern region of the grid, was used in the described tests.
The quality of the model approximation may be assessed by comparing in Figure 10 the Sigma plots
obtained for the full-system model (1,676th-order) and the reduced-order model (41st-order modal
equivalent) for the 8 x 8 $G(s)$ matrix.

The step response for the modal equivalent of a scalar transfer function model may be analytically
computed, as described in the previous section of this paper. The associated step response for the full
model may be computed by the implicit trapezoidal algorithm. The step responses for 12 of the 64 scalar
transfer functions that exist in the $G(s)$ matrix are depicted in Figure 11. Note the 41st-order model of the
scalar transfer function $g_{11}(s)$ matches rather well the response of the 1,676th-order model. The same could
not be said about the $g_{33}(s)$ and a few other scalar transfer functions, which present noticeable
discrepancies. The major system oscillations, except for one faster mode in $g_{12}(s)$ and $g_{21}(s)$, were however
all captured by the reduced-models of the twelve transfer functions.

Figure 10. Sigma-plot for 8x8 $G(s)$, $\xi = 15\%$ (reduced model has order 41).

Figure 11. Step responses for $g_{ij}(s)$ scalar transfer functions for the full model and the 41st-order model. (Note: The vertical axes in the 12 plots are given in radians/s and the horizontal axes in seconds.)

4. MODEL REDUCTION BY BALANCED TRUNCATION

This section briefly describes the model reduction of a linear multivariable dynamical system using the
balanced truncation method [14, 15, 16]. This method is then compared with the modal equivalents
alternative, considering the power system context.

Given a full-order ($n$-th order) model $G(s)$ of a system, the method’s objective is to compute a lower-
order ($r$-th order, $r < n$) model $G_r(s)$, such that the $H_\infty$ norm of the model error $e(s)=G(s)-G_r(s)$ is minimal.

Once $r$ is chosen, which involves some degree of heuristics, the method ensures there is an upper bound for
this error given by $2 \sum_{i=r+1}^n \sigma_i$, where $\sigma_i$ are the $n-r$ lowest Hankel singular values of the full system
model.

The method provides a mathematically rigorous and systematic algorithm for model order reduction. This method is widely used by the control system community for mid-sized systems and leads to very good results for both SISO and MIMO transfer functions. However, the heavy computational load involved in the calculation of the observability and controllability gramians plus the Hankel singular values of the full-order model makes the practical use of this method prohibitive for interconnected power system applications.

Other possible drawbacks of the optimal order reduction algorithms include:

- the original system poles are, in principle not preserved in the reduced model, unless previously
  specified;
- these algorithms require some pre-conditioning to deal with unstable poles;
- They produce results in a single shot, once the desired order of the reduced model is specified,
  and do not much benefit from the knowledge of the set of poles obtained in a previous run.
modal equivalents, on the other hand, are preferably used in an incremental manner, when new poles are added to the dominant pole spectrum (partial set) already found in previous runs of specific eigensolution methods [6, 13].

The technique used for model order reduction in section III is simple and easy to apply for large systems using efficient eigensolution algorithms. However, such a modal truncation approach suffers from relatively large error (difference between original and reduced model) compared to the ones based on the use of balancing transformation and Hankel singular values [17]. For large systems (thousands of states), the balanced truncation techniques are not practical since the explicit solution of Lyapunov equations is required. Probably a combination of the subspace based techniques [18, 19] coupled with Schur’s balanced truncation [20] might be better in those cases. Further investigations are needed in this area.

5. UNDERSTANDING THE ADVERSE IMPACTS ON VOLTAGE AND REACTIVE POWER PERFORMANCES DUE TO PSSS

This section describes a study of the adverse impact on generator terminal voltage and reactive power induced by the presence of a PSS. The system’s dynamic performance is evaluated for PSSs derived from different input signals.

5.1 A Tutorial Example Using a Second Order System

Let \( G(s) \) be a second order transfer function, as described in (3), where \( \omega_n \) is the undamped natural frequency and \( \zeta \) is the damping ratio [21, 22].

\[
G(s) = \frac{\omega_n^2 (as + 1)}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

Figure 12 presents results, produced by Matlab [5], on the performance of the second order system when its single zero (\( s_z \)) assumes different locations. This is carried out by altering the parameter \( a \) in the numerator of \( G(s) \). Three possibilities are investigated (\( s_z = -0.1, s_z = -0.05 \) and \( s_z = -2.0 \)), all having the same pair of complex poles. The undamped natural frequency \( \omega_n \) was set to 7 rad/s and the damping ratio \( \zeta \) to 30%.

Figure 12 shows that in all three cases the settling values are identical (equal to 1 for a step disturbance) and the settling times are practically the same (1.8s). Despite having practically no impact on these two performance indices, the zero location significantly impacted the magnitude of the transient response peak, which varied from 3.05 to 47.50. These results clearly show that the closer the zero is to the origin of the complex plane, the greater is the peak value of the system step response.

Figure 12. Time and frequency domain results for second order system.
5.2 PSS Derived from Rotor Speed or Terminal Power

The test system is a Single Machine – Infinite Bus (SMIB) system having generator and AVR models as the Xingo power plant (owned by CHESF, a major Brazilian utility). The performances of fictitious PSSs, derived from either rotor speed or terminal power, are compared. Their tuning are such that the pole associated with the electromechanical oscillation mode is placed at exactly the same location in the complex plane (Table I). This ensures that PSSs derived from different input variables may be compared on a common basis.

<table>
<thead>
<tr>
<th>Input Signal</th>
<th>PSS(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Speed</td>
<td>$(8.17) \left( \frac{1 + 0.174s}{1 + 0.010s} \right)^2 \frac{3s}{1 + 3s}$</td>
</tr>
<tr>
<td>Terminal Power</td>
<td>$(-0.26) \left( \frac{1 + 0.153s}{1 + 0.129s} \right) \frac{3s}{1 + 3s}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input Signal</th>
<th>Electromechanical Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Speed</td>
<td>$\lambda = -1.065 \pm j7.02 \ \zeta = 15%$</td>
</tr>
<tr>
<td>Terminal Power</td>
<td>$\lambda = -1.065 \pm j7.02 \ \zeta = 15%$</td>
</tr>
</tbody>
</table>

Figure 13 shows the pole-zero maps of the transfer functions (TF) that relate the active and reactive terminal power with the generator mechanical power: $(\Delta P_T/\Delta P_{MEC})$ and $(\Delta Q_T/\Delta P_{MEC})$. The pole associated with the electromechanical mode is in the same location for both stabilizers, as this was a PSS design constraint (Table I and Figure 13). The $(\Delta P_T/\Delta P_{MEC})$ TF zeros are seen in Figure 13 to have practically the same locations for both PSSs. The $(\Delta Q_T/\Delta P_{MEC})$ TF zeros, however, are seen to have considerably different locations for the two PSSs.

It is worth noting in Figure 13, which, for the $(\Delta Q_T/\Delta P_{MEC})$ TF, a zero is located near the origin of the complex plane when the PSS derived from terminal power is used. As learned from the second order tutorial example, the closer the zero is to the origin, the bigger will be the peak value for the system step response. Therefore, the worse performance of the PSS derived from terminal power can be anticipated from the analysis of the pole-zero maps depicted in Figure 13.

Figure 14 depicts step responses that allow comparing the performances of the two PSSs for a mechanical power step change (0.01 pu). The active power transient performance is the same regardless of the input signal used by the PSS. On the other hand, the reactive power transients show much bigger amplitude when the PSS derived from terminal power is used.

Note that in Figure 13 the $(\Delta Q_T/\Delta P_{MEC})$ TF is non-minimum phase, due to the presence of the right-hand-side zero. As a result, the reactive power response depicted in Figure 14, shows a transient peak of reverse polarity.

The use of electrical power as the input signal to the stabilizer, despite its simplicity, has been mostly abandoned due to its comparatively poor performance. The large transients in terminal voltage and reactive power output, shown in the simulation results of Figure 14, have usually been observed in practice following rapid changes in active power generation. Also water turbulence (pulsating torques below 0.5 Hz) in Francis turbines have yielded unacceptable sustained oscillations in terminal voltage and reactive
Terminal Active Power

Terminal Reactive Power

Terminal Voltage

Figure 14. System performance following a step disturbance on generator mechanical power.

power. A third practical disadvantage of the PSS derived from terminal power is the risk of PSS output saturation when the generator is ramping-up power, with the consequent loss of oscillation damping action.

The PSS derived from rotor speed has practical problems of another nature, associated with high-frequency noise in the speed signal. Nowadays, the preferred power system stabilizer structure is the integral of accelerating power, incorporating a fourth or higher order ramp-tracking filter in the loop. A detailed analysis of this PSS, in terms of pole-zero maps, is described in [23].

6. MODAL ANALYSIS IN POWER SYSTEM HARMONICS

In electromechanical stability studies, the modeling emphasis is given to the system electrical machines and associated controllers. The electrical network is modeled by algebraic equations, a valid assumption since the electromechanical oscillations lie in the 0.2 to 2 Hz range, which is much lower than the natural resonant frequencies of the electrical network.

Modal analysis produces valuable information on any linearized dynamic system that would otherwise be difficult to obtain by time domain or frequency response methods alone. In the case of electrical networks, some of this information is the natural oscillation modes (network resonances), identification of components that participate most in these resonances, impact of a given oscillation mode in the transient response, sensitivities to parameter changes, etc.

Despite all these advantages, modal analysis has been only moderately used in electrical network dynamic studies. This fact may be associated with the difficulties faced when using conventional state space techniques for modeling the dynamics of large RLC networks of generic topology. Recent papers have proposed the use of the descriptor system approach [24, 29] and the system nodal admittance matrix in the s-domain, $Y(s)$ [25, 26, 27, 28], to model the electrical network. These two approaches allow for an easy modeling of the electrical network in harmonic and electromagnetic transient studies. A harmonic analysis example extracted from [29] is reproduced below.

Consider the 3-bus test system shown in Figure 1, whose nominal frequency is 50 Hz and values for its lumped RLC elements are given in [29]. The symbols $I_{h1}$, $I_{h2}$ and $I_{h3}$ denote harmonic current sources connected to buses 1, 2 and 3, respectively.

Figure 15. System modeling.
The frequencies (imaginary parts divided by $2\pi$) of the complex conjugate poles (parallel resonance) and zeros (series resonance) of the network bus self-impedances, as well as their sensitivities [29] with respect to the system inductances and capacitances are presented in Table II. The sensitivities are normalized, being given in Hz/per unit change of the nominal parameter value.

**Table II. Resonance frequencies and sensitivities**

<table>
<thead>
<tr>
<th>System poles</th>
<th>Zeros seen from</th>
<th>Bus 1</th>
<th>Bus 2</th>
<th>Bus 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$(Hz)</td>
<td>$L_{CC}$</td>
<td>$-101$</td>
<td>$-11$</td>
<td>$-50$</td>
</tr>
<tr>
<td></td>
<td>$L_3$</td>
<td>$-3$</td>
<td>$-4$</td>
<td>$-2$</td>
</tr>
<tr>
<td></td>
<td>$L_4$</td>
<td>$-4$</td>
<td>$-2$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$L_{12}$</td>
<td>$-2$</td>
<td>$-78$</td>
<td>$-247$</td>
</tr>
<tr>
<td></td>
<td>$L_{13}$</td>
<td>$-19$</td>
<td>$-151$</td>
<td>$-78$</td>
</tr>
<tr>
<td></td>
<td>$C_1$</td>
<td>$-45$</td>
<td>$-12$</td>
<td>$-206$</td>
</tr>
<tr>
<td></td>
<td>$C_2$</td>
<td>$-25$</td>
<td>$-116$</td>
<td>$-111$</td>
</tr>
<tr>
<td></td>
<td>$C_3$</td>
<td>$-54$</td>
<td>$-114$</td>
<td>$-28$</td>
</tr>
</tbody>
</table>

Assume that current sources $I_{h1}$ and $I_{h3}$, shown in Figure 15, have negligible moduli while source $I_{h2}$ contains significant 5th and 11th harmonic components. The impedance values seen from bus 2 at frequencies of 250 Hz (5th harmonic) and 550 Hz (11th harmonic) are 44.48 Ω and 36.24 Ω, respectively. Thus, the 5th and 11th harmonic voltage distortions at bus 2 will be given by (4) where $I_5$ and $I_{11}$ represent the amplitudes of the 5th and 11th harmonic components, respectively.

$$V_5 = 44.48 I_5 \text{ and } V_{11} = 36.24 I_{11}$$

(4)

Assume that these distortions have exceeded their individual limits. A possible solution consists in shifting the zeros associated with the self-impedance of bus 2, located at 332 Hz and 633 Hz to the frequencies of 250 Hz and 550 Hz, respectively. The information on sensitivities of self-impedance zeros to parameter changes, contained in Table II, indicates that the desired shifts would be best accomplished by increasing the values of $C_1$ and $C_3$. Note that changing capacitor sizes is one of the recommended remedial actions against harmonic distortions [30].

Applying a Newton-Raphson algorithm based on eigenvalue sensitivity coefficients required only four iterations to obtain an accurate solution to the problem. The frequency response diagram of the bus 2 self-impedance is shown in Figure 16 for the original ($C_1 = 23.9 \mu F$ and $C_3 = 11.9 \mu F$) and new values of $C_1$ and $C_3$ ($C_1 = 29.26 \mu F$ and $C_3 = 22.77 \mu F$). The new frequency values of the poles and zeros are presented in Table III.

**Table III. Frequencies of poles and zeros for the new values of capacitances at bus 1 and 3**

<table>
<thead>
<tr>
<th>System poles</th>
<th>Zeros seen from</th>
<th>Bus 1</th>
<th>Bus 2</th>
<th>Bus 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$(Hz)</td>
<td>$L_{CC}$</td>
<td>$-101$</td>
<td>$-11$</td>
<td>$-50$</td>
</tr>
<tr>
<td></td>
<td>$L_3$</td>
<td>$-3$</td>
<td>$-4$</td>
<td>$-2$</td>
</tr>
<tr>
<td></td>
<td>$L_4$</td>
<td>$-4$</td>
<td>$-2$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$L_{12}$</td>
<td>$-2$</td>
<td>$-78$</td>
<td>$-247$</td>
</tr>
<tr>
<td></td>
<td>$L_{13}$</td>
<td>$-19$</td>
<td>$-151$</td>
<td>$-78$</td>
</tr>
<tr>
<td></td>
<td>$C_1$</td>
<td>$-45$</td>
<td>$-12$</td>
<td>$-206$</td>
</tr>
<tr>
<td></td>
<td>$C_2$</td>
<td>$-25$</td>
<td>$-116$</td>
<td>$-111$</td>
</tr>
<tr>
<td></td>
<td>$C_3$</td>
<td>$-54$</td>
<td>$-114$</td>
<td>$-28$</td>
</tr>
</tbody>
</table>

For these new values of $C_1$ and $C_3$ the impedance values seen from bus 2 at the frequencies of 250 Hz and 550 Hz (5th and 11th harmonics, respectively) have changed from 44.48 Ω to 12.73 Ω and from 36.24 Ω to 2.81 Ω. This means a reduction in voltage distortion at bus 2 of 70 % at the 5th and 90 % at the 11th harmonics. It must be pointed out that the zeros can be shifted without changing the system operating point at fundamental frequency (maintaining the original power flow condition) as described in [29].
Figure 16. Self-impedance seen from bus 2 for both the original and the new values of capacitances at buses 1 and 3.

7. FINAL REMARKS

This paper described applications of modal analysis and other linear techniques to electromechanical oscillation damping and harmonic problems of large electrical networks. The dynamic information provided by modal analysis is highly useful and complementary to those obtained by other methodologies. A well-designed graphical user interface and visualization of results allows a more effective utilization of the small-signal stability package [31, 32].

Additional work carried out by the authors, in modal analysis and related topics, are listed below:

- Partial eigensolution algorithms [6, 13, 33];
- Frequency response [34, 35];
- Modal sensitivities and mode shapes (transfer function residues, controllability and observability factors and participation factors) [12, 31];
- Root-locus and root-contour plots [2];
- Coordinated tuning of damping controllers [36];
- Hopf bifurcations [37];
- Assessing PSS robustness through a synthetic system with variable impedance [38];
- Subsynchronous resonance studies [39];
- Modeling power systems directly in the s-domain [26, 27, 28];
- Small-signal voltage stability [40];
- Electromagnetic transients [28].

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